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A probabilistic theory of the strength of short-fibre composites with variable fibre length and orientation

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This paper adopts a probabilistic approach to examine the effects of fibre length and orientation distribution on the strength of short fibre composites. A general theory has been formulated in terms of fibre length and orientation distribution function as well as the composite geometrical and physical properties. The final result is presented in the form of a modified "rule of mixtures". The result has been applied to discuss several special cases of fibre arrangements. They are (a) unidirectional short fibre composites with uniform fibre length, (b) unidirectional short fibre composites with fibre length distribution, (c) random short fibre composites with uniform fibre length and (d) partially-aligned short fibre composites with uniform fibre length. Comparisons of the present results with previous work are also discussed.

1. Introduction

Discontinuous fibre-reinforced composite materials have versatile properties and are relatively inexpensive to fabricate. In a typical injection moulded material, for instance, the fibres are relatively short, variable in length and imperfectly aligned. As discussed by Chou and Kelly [1, 2], the strength and failure behaviour of short fibre composites are complicated by the non-uniformity in fibre length and orientation.

The "rule of mixtures" is often used to predict the strength of fibre-reinforced composites. For unidirectional continuous fibre composites, under the assumption of isostrain in the fibres and matrix, the rule of mixtures becomes [3, 4]

$$\sigma_{cu} = \sigma_{fu} V_f + \sigma'_m (1 - V_f), \qquad (1)$$

where σ_{cu} and σ_{fu} are the ultimate strength of the composite and fibre, respectively, V_f denotes the fibre volume-fraction, and σ'_m is the matrix stress at the failure of the composite.

In the case of unidrectional short fibre composites, Equation 1 is modified as follows [3, 4]

$$\sigma_{cu} = \sigma_{fu} V_f F(l_c/\bar{l}) + \sigma'_m (1 - V_f), \qquad (2)$$

Here, the factor $F(l_c/\bar{l})$ is added to take into account the effect of fibre length, and l_c and $\bar{l}$ denote fibre critical length and average length, respectively. If a constant interfacial shear stress and a uniform fibre length, $\bar{l}$, are assumed, this factor becomes

$$F(l_c/\bar{l}) \begin{cases} = 1 - l_c/2\bar{l} & (\bar{l} > l_c) \\ = \bar{l}/2l_c & (\bar{l} < l_c). \end{cases} \qquad (3)$$

If the fibre length is not uniform, Equation 3 must be modified and this problem is the first objective of this paper. It should be noted at this point that Riley [5] and Fukuda and Chou [6] have also dealt with the strength of unidirectional short fibre composites. Riley took into consideration the disturbance of fibre stress due to the presence of a fibre end. Fukuda and Chou advanced Riley's idea to a general case by introducing the theory of probability. However, in the present paper, we focus on the geometrical arrangement of fibres and the effect of stress redistribution is not considered.

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In actual short fibre composites, such as injection moulded materials or sheet moulding compounds, however, there are variations not only in fibre length but also in fibre orientation. In cases when the fibres are misaligned, the rule of mixtures is further modified to [7, 8]

$$\sigma_{cu} = \sigma_{fu} V_f F(l_c/\bar{l}) C_0 + \sigma'_m (1 - V_f), \qquad (4)$$

where the fibre orientation factor, C_0, has hitherto been determined by experiments [7, 8] and there has been no theory to predict this value. However, if all necessary conditions with respect to the fibre orientation are known, C_0 can be estimated through a theoretical analysis. This is another goal of the present paper. Fukuda and Kawata [9] derived the "coefficient of alignment", C_a, which they used to predict the elastic modulus of misaligned short fibre composites. Although C_0 is somewhat different from C_a, the basic probabilistic approach described in [9] is still useful and it has been adopted in the present analysis.

Bader, Chau and Quigly [10], and Fukuda and Chou [6] proposed the concept of "critical zone" in predicting the strength of unidirectional short fibre composites. The critical zone is defined by a pair of planes separated by a distance $\beta\bar{l}$, where β is a constant less than 1 and $\bar{l}$ is the average fibre length, which cut across the section normal to the applied tensile stress and fibre alignment direction. They assumed that fibres which do not "bridge" the zone cannot contribute to the strength of the zone. This concept is expanded in the present paper to a problem with variable fibre orientation and length. It will be shown that a unidirectional fibre model is a special case of the present general model.

In order to deal with the distributions of fibre length and fibre orientation, we have introduced two kinds of probability density functions. Then the composite strength, which is the ultimate goal of the research, should be given in the form of a probability density function. However, it is understood that in the present analysis the composite strength is derived only in the form of an average value.

2. General theory

2.1. Geometrical consideration on a single short fibre

First, the geometrical arrangement of one short fibre is described. Fig. 1 shows an obliquely positioned short fibre of length, l. In accordance with the terminology of [10], a bridging fibre and an ending fibre are defined as shown in Fig. 1b; that is, if a fibre crosses a critical zone of width $\beta\bar{l}$, it is called a bridging fibre, and if the end of a fibre is within the critical zone, it is called an ending fibre. The probability density function of fibre length distribution, $h(l)$, satisfies the following condition

$$\int_0^\infty h(l)\mathrm{d}l = 1. \qquad (5)$$

Then, the average fibre length is defined as

$$\bar{l} = \int_0^\infty l h(l)\mathrm{d}l. \qquad (6)$$

From Fig. 1a,

$$l_z = l\cos\theta. \qquad (7)$$

and from Fig. 1c the critical angle, θ_0, within which a fibre of length l is a bridging fibre becomes

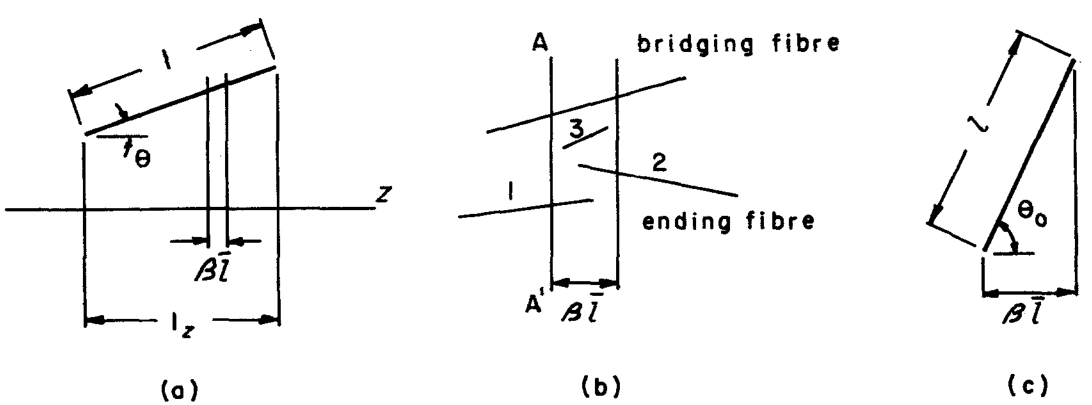

Figure 1 Several notations on short fibre arrangement. (a) obliquely oriented fibre, (b) bridging fibre and ending fibre, (c) critical angle.

$$\theta_0 = \cos^{-1} \beta\bar{l}/l \tag{8}$$

for $\beta\bar{l} \leqslant l$. If $\beta\bar{l} > l$, θ_0 cannot be defined, and a fibre in such a case is inevitably an ending fibre. If the fibres are distributed randomly with respect to the z-azis, the probability, p_e, that a fibre of length, l, and orientation angle, θ, is an "ending fibre" in the critical zone becomes

$$p_e = \frac{\beta\bar{l}}{l_z} = \begin{cases} \beta\bar{l}/l \cos\theta & (0 \leqslant \theta \leqslant \theta_0 \text{ and } \beta\bar{l} \leqslant l) \\ 1 & (\theta_0 \leqslant \theta \leqslant \pi/2 \text{ or } \beta\bar{l} \geqslant l), \end{cases} \tag{9}$$

and the probability, p_b, for finding a bridging fibre is, by definition,

$$p_b = 1 - p_e. \tag{10}$$

By assuming the distribution of the fibre orientation to be symmetrical with respect to the applied tensile load along the z-axis, the probability density functions with respect to fibre orientation, $g(\theta)$, should satisfy the condition

$$\int_0^{\pi/2} g(\theta)\mathrm{d}\theta = 1. \tag{11}$$

2.2. Load transfer in short fibre

First, a short fibre situated parallel to the applied tensile stress, σ_0, is considered, as shown in Fig. 2a. The average fibre stress is

$$\sigma_{f0} = \frac{1}{l}\int_0^l \sigma_f(z)\mathrm{d}z. \tag{12}$$

Although there are several analyses on the fibre stress distribution [11–13], the simplest model [3, 4] is applied here, as shown in Figs 2b and c. This model assumes that a constant interfacial shear stress acts in the regions of $0 \leqslant z \leqslant l_c/2$ and $l - l_c/2 \leqslant z \leqslant l$. Then, σ_{f0} becomes

$$\sigma_{f0} = \begin{cases} \sigma_{fu}\left(1 - \dfrac{l_c}{2l}\right) & (l > l_c) \\ \sigma_{fu}\dfrac{l}{2l_c} & (l < l_c) \end{cases} \tag{13}$$

The average force in a fibre is $\sigma_{f0}A_f$, where A_f is the fibre cross-sectional area.

Next, a single short fibre situated at an angle, θ, to the applied stress is considered. From the equilibrium of applied force, the model of Fig. 3a is equivalent to that of Fig. 3b where

$$\sigma_0' = \sigma_0 \cos^2\theta \tag{14}$$

and

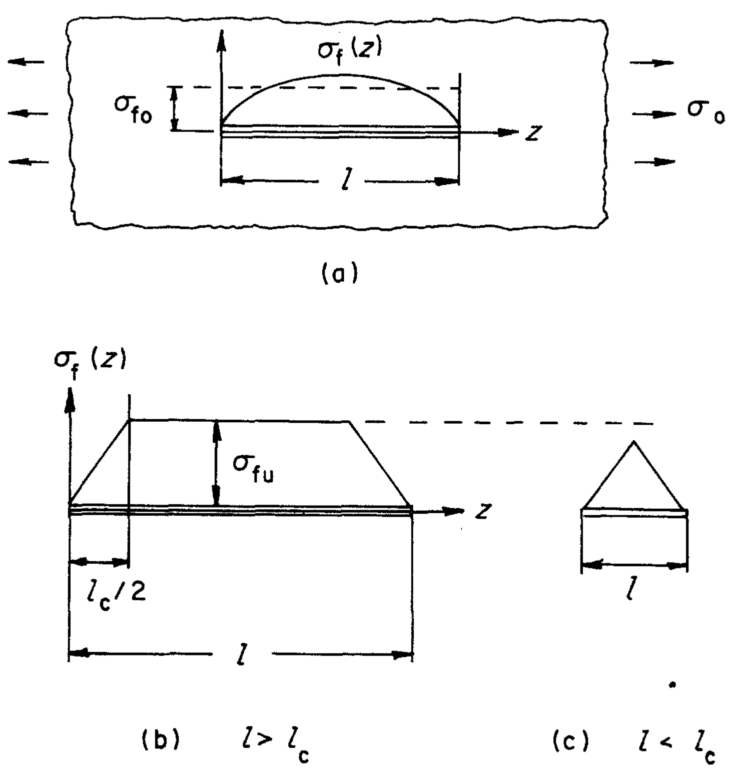

Figure 2 Stress distribution of a fibre parallel to the applied tensile stress. (a) actual stress distribution, (b) and (c) simplified stress distribution.

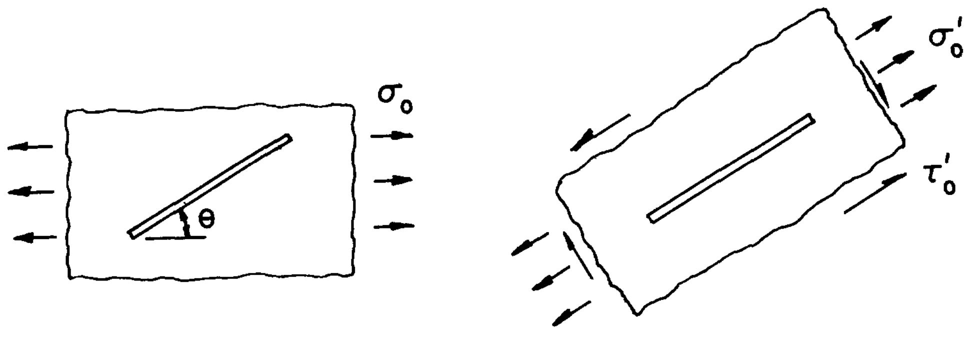

Figure 3 Applied stress to an obliquely oriented fibre. (b) is equivalent to (a).

$$\tau_0' = \sigma_0 \sin\theta \cos\theta, \tag{15}$$

where τ_0' is the shear stress. If the effect of τ_0' on the fibre stress distribution can be neglected, the argument concerning Fig. 2 can be applied to an obliquely oriented fibre model by changing only σ_0 to $\sigma_0' \cos^2\theta$. Then the average force of the fibre becomes $A_f\sigma_{f0}\cos^2\theta$ and the z-direction force component is

$$F_z = A_f\sigma_{f0}\cos^3\theta. \tag{16}$$

2.3. Strength of short fibre composites

Based upon the above preparations, the strength of short fibre composites can now be derived. In the following discussion, $h(l)$ and $g(\theta)$ are assumed to be independent of each other. This means that $g(\theta)$ is the same for all the samples with different fibre length distributions. Suppose a rectangular-shaped specimen with the lengths of the three mutually perpendicular edges denoted by a, b and c is considered. The c-axis is so chosen as to be parallel to the z-axis. The volume of the specimen is then

$$V = abc, \tag{17}$$

and from the definition of fibre volume-fraction, V_f becomes

$$V_f = NA_f\bar{l}/V, \tag{18}$$

where N and A_f denote, respectively, the total number of fibres and the fibre cross-sectional area.

Recall that Equation 7 gives the length of the projection of a fibre on the z-axis. Then the average length of the projection of fibres can be written as

$$\bar{l}_z = \int_0^{\pi/2}\int_0^{\infty} l\cos\theta\, h(l)\, g(\theta)\, dl d\theta$$

$$= \bar{l}\int_0^{\pi/2} g(\theta)\cos\theta\, d\theta. \tag{19}$$

The value $N\bar{l}_z$ gives the total length of projection of all the fibres on the z-axis and if this value is divided by the specimen length, c, this gives the average number of fibres which cross an arbitrary section in the specimen normal to the z-axis. That is,

$$N_c = \frac{N\bar{l}_z}{c} = \frac{abV_f}{A_f}\int_0^{\pi/2} g(\theta)\cos\theta\, d\theta. \tag{20}$$

Equation 9 is the probability of a specific fibre being an ending fibre. Therefore, based upon the assumption of constant fibre volume-fraction in any critical zone, the average value of probability of finding an arbitrary fibre being an ending fibre is

$$q_e = \int_0^{\pi/2}\int_0^{\infty} p_e h(l)\, g(\theta)\, dl d\theta. \tag{21}$$

Similarly, the average value of the probability of finding an arbitrary fibre to be a bridging fibre is

$$q_b = \int_0^{\pi/2}\int_0^{\infty} p_b h(l)\, g(\theta)\, dl d\theta$$

$$= 1 - q_e. \tag{22}$$

Substituting Equations 9 and 10 into Equations 21 and 22 gives

$$q_e = \int_0^{\theta_0} d\theta\left[\int_0^{\beta\bar{l}} g(\theta)\, h(l)\, dl + \int_{\beta\bar{l}}^{\infty} \frac{\beta\bar{l}}{l\cos\theta} g(\theta)\, h(l)\, dl\right]$$

$$+ \int_{\theta_0}^{\pi/2}\int_{\beta\bar{l}}^{\infty} g(\theta)\, h(l)\, dl d\theta \tag{23}$$

and

$$q_b = \int_{\beta\bar{l}}^{\infty} dl \int_0^{\theta_0} \left(1 - \frac{\beta\bar{l}}{l\cos\theta}\right) g(\theta)\, h(l)\, d\theta. \quad (24)$$

Then, the total numbers of ending and bridging fibres in the specimen are

$$N_e = N_c q_e \quad (25)$$

and

$$N_b = N_c q_b. \quad (26)$$

Strictly speaking, the value of N_e is not precise because we have only examined one cross-section, for example, AA′ in Fig. 1b; the fibres denoted 2 and 3 in Fig. 1b were not considered. However, our interest is to calculate the number of bridging fibres, which is not affected by N_e in the subsequent discussions.

Based upon Equation 16 for the z-direction component of the axial load of one specific fibre, the average value among the bridging fibres is

$$\bar{F}_z = \int_0^{\theta_0}\int_{\beta\bar{l}}^{\infty} F_z h(l)\, g(\theta)\, dl d\theta. \quad (27)$$

Then the total load that all of the bridging fibres can sustain in the zone $\beta\bar{l}$ is

$$F_T = N_b \cdot \bar{F}_z \quad (28)$$

and the composite strength becomes

$$\sigma_{cu} = \frac{F_T}{ab} + \sigma'_m (1 - V_f). \quad (29)$$

where the matrix is assumed to sustain part of the applied load. Substituting Equations 13, 16, 20 and 23 to 28 into Equation 29, we finally obtain

$$\sigma_{cu} = \sigma_{fu} V_f \int_{\theta_0}^{\pi/2} g(\theta)\cos\theta\, d\theta \int_0^{\theta_0} g(\theta)\cos^3\theta\, d\theta$$
$$\times \int_{\beta\bar{l}}^{\infty} \left[\int_0^{\theta_0} \left(1 - \frac{\beta\bar{l}}{l\cos\theta}\right) g(\theta)\, d\theta\right] h(l)\, dl$$
$$\times \left[\int_{\beta\bar{l}}^{l_c} \frac{l}{2l_c} h(l)\, dl + \int_{l_c}^{\infty} \left(1 - \frac{l_c}{2l}\right) h(l)\, dl\right]$$
$$+ \sigma'_m (1 - V_f). \quad (30)$$

Equation 30 is a general strength expression of short fibre components. In order to conduct further analysis, it is necessary to know the functions $g(\theta)$ and $h(l)$ together with σ_{fu}, σ'_m, V_f and l_c.

3. Results and discussion

First, the limiting case of a unidirectional composite with uniform fibre length is considered. The condition of unidirectional fibre arrangement requires that $g(\theta)$ be a delta function at $\theta = 0$. Hence, we obtain

$$\int_0^{\pi/2} g(\theta)\cos\theta\, d\theta = 1, \quad (31)$$

$$\int_0^{\theta_0} g(\theta)\cos^3\theta\, d\theta = 1 \quad (32)$$

and

$$\int_0^{\theta_0} \left\{1 - \frac{\beta\bar{l}}{l\cos\theta}\right\} g(\theta)\, d\theta = 1 - \beta l/\bar{l}. \quad (33)$$

Therefore, Equation 30 is reduced to

$$\sigma_{cu} = \sigma_{fu} V_f \int_{\beta\bar{l}}^{\infty} (1 - \beta\bar{l}/l)\, h(l)\, dl \left[\int_{\beta\bar{l}}^{l_c} \frac{l}{2l_c} h(l)\, dl + \int_{l_c}^{\infty} \left(1 - \frac{l_c}{2l}\right) h(l)\, dl\right] + \sigma'_m (1 - V_f). \quad (34)$$

Furthermore, the condition of uniform fibre length means that $h(l)$ is a delta function at $\bar{l}$

$$\int_{\beta\bar{l}}^{\infty} (1 - \beta\bar{l}/l)\, h(l)\, dl = 1 - \beta \qquad (\text{all } \bar{l}) \quad (35)$$

and

$$\int_{\beta\bar{l}}^{l_c} \frac{l}{2l_c} h(l)\, dl + \int_{l_c}^{\infty} (1 - l_c/2l)\, h(l)\, dl = \begin{cases} 1 - l_c/2\bar{l} & (\bar{l} > l_c) \\ \bar{l}/2l_c & (l < l_c). \end{cases} \quad (36)$$

Then, Equation 34 becomes

$$\sigma_c = \sigma_{fu} V_f (1 - \beta)\left(1 - \frac{l_c}{2\bar{l}}\right) + \sigma'_m (1 - V_f) \qquad (\bar{l} > l_c) \quad (37)$$

and

$$\sigma_c = \sigma_{fu} V_f (1 - \beta)\frac{\bar{l}}{2l_c} + \sigma'_m (1 - V_f) \qquad (\bar{l} < l_c). \quad (38)$$

Equation 37 is identical to Equation 6 of [10] although different notations have been used and the second term of Equation 37 is omitted in Equation 6 of [10]. Consequently, the theory of [10] can be included in the present theory as a limiting case.

Next, the effect of fibre-length distribution on composite strength for a unidirectional fibre model is considered. We consider the limiting case of $\beta \to 0$, namely, all fibres are bridging fibres. Then, Equation 34 becomes

$$\sigma_{cu} = \sigma_{fu} V_f \left[\int_0^{l_c} \frac{l}{2l_c} h(l)\, dl + \int_{l_c}^{\infty} \left(1 - \frac{l_c}{2l}\right) h(l)\, dl\right] + \sigma'_m (1 - V_f). \quad (39)$$

The following probability density function is chosen for a case study

$$h(l) = \frac{\pi}{4\bar{l}} \sin\frac{\pi l}{2\bar{l}} \quad (0 \leqslant l/\bar{l} \leqslant 2). \quad (40)$$

This function satisfies both Equations 6 and 11. Substituting Equation 40 into Equation 39, and after some calculation, the following result is obtained:

$$\sigma_{cu} = \sigma_{fu} V_f \left\{\frac{1}{2} + \frac{\bar{l}}{2\pi l_c} \sin\frac{\pi l_c}{2\bar{l}} + \frac{1}{4}\cos\frac{\pi l_c}{2\bar{l}} - \frac{\pi l_c}{8\bar{l}}\left[\mathrm{Si}(\pi) - \mathrm{Si}\left(\frac{\pi l_c}{2\bar{l}}\right)\right]\right\} + \sigma'_m (1 - V_f), \quad (41)$$

where $\mathrm{Si}(X)$ is the integral sine function defined by

$$\mathrm{Si}(X) = \int_0^X \frac{\sin t}{t}\, dt. \quad (42)$$

The result of Equation 41 is shown as a solid in Fig. 4. In the case of constant fibre length, the strength can be obtained from Equations 2 and 3 and the value is also shown in Fig. 4 by a broken line. It can be concluded from Fig. 4 that the strength of a composite material is reduced if the fibre length is not uniform. However, the difference between the present result and the ordinary theory is not very great and, hence, the ordinary theory may be used as a first approximation.

As another example the strength prediction based upon Equation 30, we examine the orientation factor, C_0, for random array composites. The fibre length is assumed to be uniform and is larger than l_c. These assumptions lead to

$$\int_{\beta\bar{l}}^{l_c} \frac{l}{2l_c} h(l)\, dl = 0, \quad (43)$$

$$\int_{l_c}^{\infty} \left(1 - \frac{l_c}{2l}\right) h(l)\, dl = 1 - l_c/2\bar{l} \quad (44)$$

and

$$\int_{\beta\bar{l}}^{\infty} \left[\int_0^{\theta_0} \left(1 - \frac{\beta\bar{l}}{l\cos\theta}\right) g(\theta)\, d\theta\right] h(l)\, dl = \int_0^{\theta_0} \left(1 - \frac{\beta}{\cos\theta}\right) g(\theta)\, d\theta \quad (45)$$

Then Equation 30 becomes

$$\sigma_{cu} = \sigma_{fu} V_f \left(1 - \frac{l_c}{2l}\right) \int_0^{\pi/2} g(\theta)\cos\theta\, d\theta \times \int_0^{\theta_0} g(\theta)\cos^3\theta\, d\theta \int_0^{\theta_0}\left(1 - \frac{\beta}{\cos\theta}\right) \times g(\theta)\, d\theta + \sigma'_m (1 - V_f). \quad (46)$$

By comparing Equations 4 and 46, we arrive at the following expression for C_0:

$$C_0 = \int_0^{\pi/2} g(\theta)\cos\theta\, d\theta \int_0^{\theta_0} g(\theta)\cos^3\theta\, d\theta \times \int_0^{\theta_0}\left(1 - \frac{\beta}{\cos\theta}\right) g(\theta)\, d\theta. \quad (47)$$

We consider both two-dimensional and three-dimensional random arrays. In a two-dimensional random array model, $g(\theta)$ must be constant in the whole region of $0 \leqslant \theta \leqslant \pi/2$. Then we can get

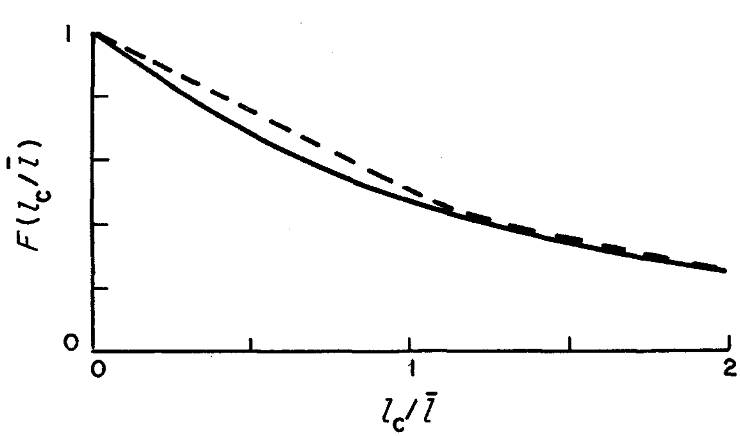

Figure 4 Effect of fibre length on composite strength, $F(l_c/\bar{l})$. Solid line: fibre distribution is considered; broken line: fibre length is assumed to be constant.

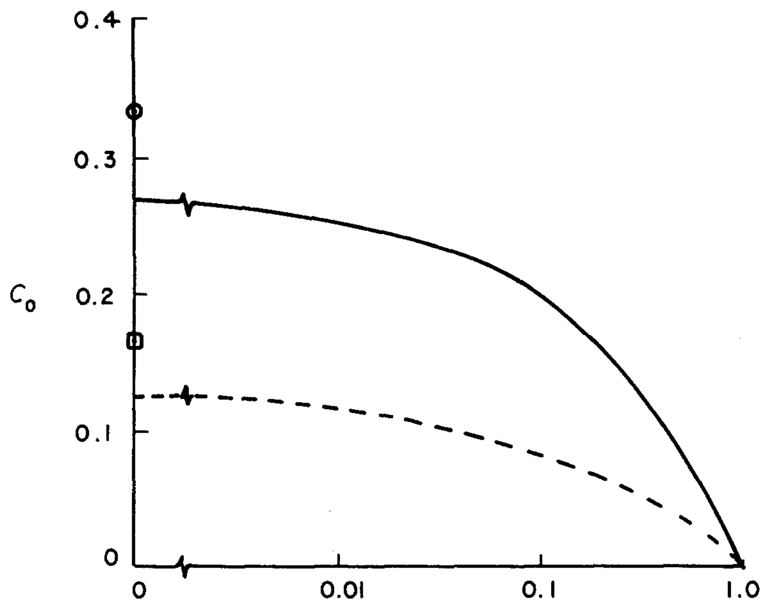

Figure 5 Fibre orientation factor, C_0, of random array model. Solid line: two-dimensional random array; broken line: three-dimensional random array. Circled dot and squared dot are Cox's result.

$$g(\theta) = 2/\pi \tag{48}$$

from Equation 11. Substituting Equation 48 into Equation 47 gives

$$C_0 = \frac{4}{\pi^2}\frac{2+\beta^2}{3}(1-\beta^2)^{1/2}\frac{2}{\pi}\left[\cos^{-1}\beta - \frac{1}{2}\beta\log\frac{1+(1-\beta^2)^{1/2}}{1-(1-\beta^2)^{1/2}}\right]. \tag{49}$$

The solid line of Fig. 5 depicts this result. At the limit of $\beta \to 0$, C_0 tends to $2/3(2/\pi)^2 = 0.270$. On the other hand, Bowyer and Bader [7] used the value of 1/3 by quoting the result of Cox [11]. However, it should be noted that Cox obtained this value in calculating the orientation factor for the Young's modulus of a random composite. Cox's value is also shown in Fig. 5 by a circled dot.

In the case of a three-dimensional random array model from Fig. 6, $g(\theta)$ can be expressed as

$$g(\theta)\,\mathrm{d}\theta = \mathrm{d}S/S \tag{50}$$

and, therefore,

$$g(\theta) = \sin\theta. \tag{51}$$

In this case, Equation 47 becomes

$$C_0 = \tfrac{1}{8}(1-\beta^2)(1+\beta^2)(1-\beta+\beta\log\beta). \tag{52}$$

This value is also shown in Fig. 5 by a broken line. At the limit of $\beta \to 0$, C_0 becomes 1/8 and this value is again a little smaller than Cox's result of

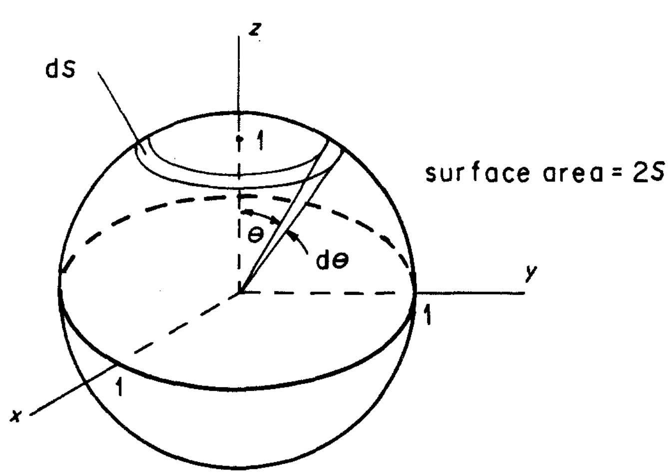

Figure 6 Some notations concerning the surface of a unit sphere.

1/6 (see Fig. 5). In both two- and three-dimensional cases, our results are smaller than those of Cox's for the effective moduli of short fibre composites. The significance of this difference is open to future discussions.

It is rare in short fibre composites to find that all fibres are aligned unidirectionally. It is also rare that fibres are randomly oriented. The present theory is now applied to the intermediate fibre orientation. In order to avoid the complexity of a general model, we again start from Equation 47, that is, the fibre length is assumed to be constant. The following two types of fibre orientations are considered:

(a) $g(\theta) = 1/\alpha$ in $0 \leqslant \theta \leqslant \alpha$ and $g(\theta) = 0$ in $\theta > \alpha$;

(b) $g(\theta) = \pi/2\alpha \cos \pi\theta/2\alpha$ in $0 \leqslant \theta \leqslant \alpha$ and $g(\theta) = 0$ in $\theta > \alpha$.

These functions are taken so as to satisfy Equation 11 and the shapes of these functions are shown schematically in Fig. 7. Note that $g(\theta)$ does not mean the probability per unit area. The probability per unit area is proportional to $g(\theta)/\sin\theta$. The limit of $\beta \to 0$ is again considered. At this limit, θ_0 tends to $\pi/2$ from Equation 8. Considering this condition, C_0 is calculated from Equation 26 for the two types of $g(\theta)$ given above.

For Type a

$$\lim_{\beta\to 0} C_0 = \frac{\sin\alpha}{\alpha}\frac{1}{\alpha}\left(\frac{1}{12}\sin 3\alpha + \frac{3}{4}\sin c\right). \quad (53)$$

For Type b

$$\lim_{\beta\to 0} C_0 = \frac{1}{16}\left[\frac{1}{1+q}\sin\frac{\pi}{2}(1+q) + \frac{1}{1-q}\sin\frac{\pi}{2}(1-q)\right]\left[\frac{3}{1+q}\sin\frac{\pi}{2}(1+q) + \frac{3}{1-q}\sin\frac{\pi}{2}(1-q) + \frac{1}{1+3q}\sin\frac{\pi}{2}(1+3q) + \frac{1}{1-3q}\sin\frac{\pi}{2}(1-3q)\right], \quad (54)$$

where $q = 2\alpha/\pi$. These values are shown in Fig. 7. Bower and Bader [7] estimated the value of C, which corresponds to C_0 of the present paper, by their experimental data. For laboratory glass–nylon injection-moulded materials, C_0 was 0.66. If we use a rectangular distribution for $g(\theta)$, the value α, corresponding to $C_0 = 0.66$ is approximately 45° from Fig. 7. Although the distribution

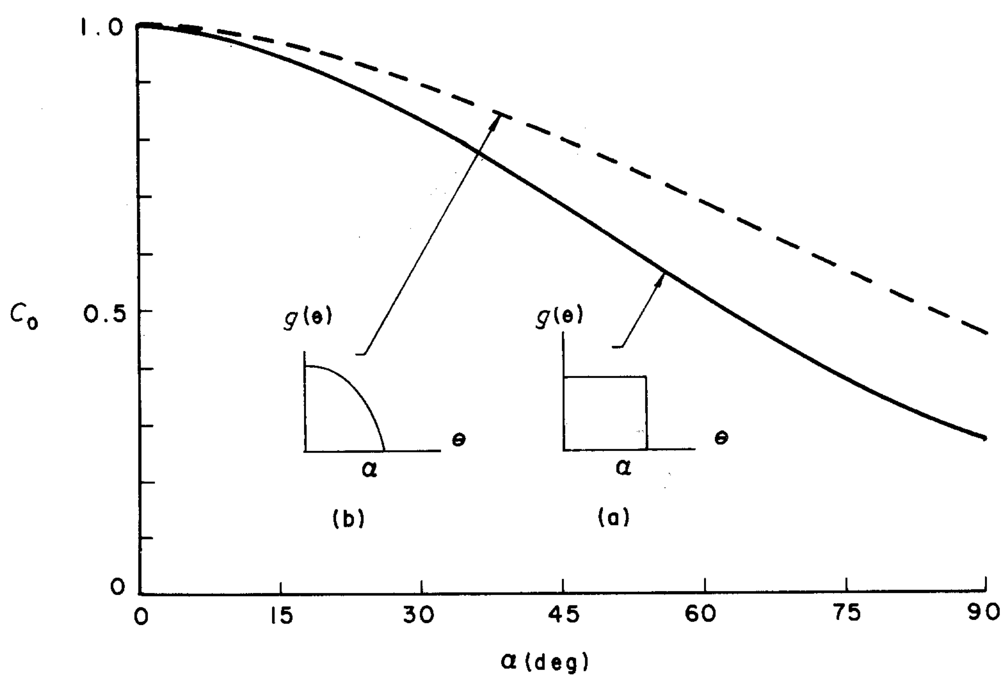

Figure 7 Value of C_0 for two types of distributions of fibre orientation.

of fibre orientation is not reported in [7]. Fig. 1 of [7] provides us with some useful information.

Fig. 1 of [7] shows the longitudnal section through an injection-moulded glass–nylon specimen. If a fibre is inclined at an angle, θ, from the longitudinal axis of the specimen, an ellipse of the obliquely cut section of the fibre will appear in the figure. The ratio of the major axis to the minor axis of this ellipse is $1/\sin\theta$. If we can determine $g(\theta)$ from specimen cross-sections, C_0 can be calculated using Equation 30 in the general case or Equation 47 in the case of uniform fibre length. We have taken the reversed route, that is, knowing the C_0 value, the scattering of fibre orientation was estimated under the assumption of $g(\theta)$ being rectangular. Although further quantitative discussions are difficult at the present stage because of the lack of rigorous experimental information, our theory is useful for estimating C_0.

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